Title: Understanding Limits, Continuity, and Differentiability

Subtitle: Exploring Fundamental Concepts in Calculus

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Introduction

- Brief introduction to the fundamental concepts of calculus: limits, continuity, and differentiability.
- Importance of these concepts in understanding the behaviour of functions and solving problems in calculus.

Outline:

1. Introduction to Limits
2. Understanding Continuity
3. Exploring Differentiability
4. Lagrange Mean Value Theorem
5. Rolle's Theorem
6. Conclusion

Definition of Limit

- Definition: Let $f(x)$ be a function defined around $x=a$, except possibly at $x=a$. We say that the limit of $f(x)$ as x approaches a is L, denoted by $\lim_{x\to a} f(x)=L$, if for every $\epsilon>0$, there exists $\delta>0$ such that if $0<|x-a|<\delta$, then $|f(x)-L|<\epsilon$.
- Notation: $\lim_{x\to a} f(x)= L$
- Examples:

1.Find lim $\lim_{x\to 2} (3x-1)$.

2.Evaluate $\lim_{x\to 0} x\sin(x)$.

Definition of Continuity

- Definition: A function $f(x)$ is continuous if $\lim_{x \to a} f(x) = f(a)$.
- Types of discontinuities: Removable, jump, and infinite discontinuities.
- Examples:

 1. Investigate the continuity of $g(x) = x$ at $x = 0$.

Definition of Differentiability

- A function $f(x)$ is said to be differentiable at a point $x=a$ if the derivative of $f(x)$ exists at $x=a$.
- The derivative of $x=a$, denoted as $f'(a)$, is defined as:

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \text{, exist}$$

- *And* $L(f^1(a)) = R(f^1(a))$

Example -

1.Check the differentiability of $f(x)=|x|$ at $x=0$.

Lagrange Mean Value Theorem (LMVT)

- **Statement**: If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b), then there exists c in (a,b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$
- Geometric interpretation.
- **Examples:**

  1. Verify LMVT for $f(x)=x^2$ on $[1,3]$.
  2. Apply LMVT to $g(x)=\ln(x)$ on $[1,e]$.
  3. Use LMVT to find a point c for $h(x)=\sin(x)$ on $[0,\pi]$.

Rolle's Theorem

- **Statement**: If $f(x)$ is continuous on $[a,b]$, differentiable on (a,b), and $f(a)=f(b)$, then there exists c in (a,b) such that $f'(c)=0$.
- Geometric interpretation.
- **Examples:**

1. Show that $f(x)=x^3-3x^2+2x$ satisfies Rolle's Theorem on $[-1,2]$.
2. Apply Rolle's Theorem to $g(x)=\sin(x)$ on $[0,\pi]$.
3. Investigate the conditions for $h(x)=\cos(x)$ to satisfy Rolle's Theorem.

Conclusion

- Summary of the key points covered in the presentation.
- Importance of understanding limits, continuity, and differentiability in calculus.
- Encouragement for further exploration and practice.

THANK YOU